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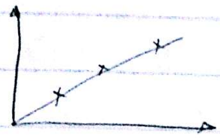
Numerical Analysis
Ch 1 : Curve Fitting

Least mean method : هى طريقة توفيق المنحنيات باستخدام خط مستقيم

$y = a + bx$ $\qquad$ $y = ax + b$



$$y_0 = a x_0 + b$$
$$y_1 = a x_1 + b$$
$$\vdots$$
$$y_n = a x_n + b$$

$\longrightarrow \sum y = a \sum x + nb \qquad (1)$

$\longrightarrow \sum xy = a \sum x^2 + b \sum x \qquad (2)$

مقدار الخطأ

"بعد النقطة الفعلية عن الخط اللى احنا جبناه" $D = |y_i - y_{iappl}|$

القيمة الفعلية

القيمة التقريبية المناظرة للقيمة الحقيقية على الخط

مقدار الخطأ الكلى $RMSE = \sqrt{\dfrac{\sum D^2}{n}}$

ideas

1 – $y = ax^b$

نعمل linearalization ← نستخدم ln

$$\ln y = \ln a + b \ln x$$

putting $\longrightarrow \ln y = Y$ & $\ln x = X$ & $A = \ln a$

$$\therefore Y = A + bX$$

2 – $y = a + bx^c$ ← قيمة معطاة

let $x^c = X$ $\longrightarrow$ ثم نكمل عادى

3 – $y = \dfrac{1}{a + b\cos\theta}$

$\dfrac{1}{y} = a + b\cos\theta \longrightarrow \dfrac{1}{y} = Y$ & $\cos\theta = X$

$$\therefore Y = a + bX$$

4_ $y = \dfrac{x}{ax + b}$

$\dfrac{1}{y} = \dfrac{ax+b}{x} \longrightarrow \dfrac{1}{y} = a + b\left(\dfrac{1}{x}\right)$

$\therefore Y = a + bX$

5_ $y = ab^x$

$\ln y = \ln a + x \ln b$

$Y = A + BX$

$a = e^A \quad \& \quad b = e^B$

6_ $y = ae^{bx}$

$\ln y = \ln a + bx \cancel{\ln e}$

$\therefore Y = A + bx \longrightarrow a = e^A$

① Fit the curve $R = at + b$ and find $f(40)$ to the reading

t	30.5	35.4	50.2	78.1	90.5
R	610	708	810	1562	1900

solution

i	t	R	tR	t^2
1	30.5	610		
2	35.4	708		
3	50.2	810		
4	78.1	1562		
5	90.5	1900		
Σ	284.7	5590	378272.4	18993.3

$\therefore R = at + b$

$\Sigma R = a\Sigma t + nb \longrightarrow 5590 = 284.7a + 5b$

$\Sigma Rt = a\Sigma t^2 + b\Sigma t \longrightarrow 378272.4 = 18993.31a + 284.7b$

بالآلة الحاسبة $\longrightarrow a = 21.56 \quad b = 109.366$

$\therefore R = 21.56t + 109.366$

$R_{40} \cong 21.56(40) + 109.366$

② Fit the curve $y = \frac{1}{a + b\cos\theta}$

θ	30	4.5	60
y	0.225	0.27	0.32

solution $\frac{1}{y} = a + b\cos\theta$

$\therefore Y = a + bX$

i	θ	y	$X = \cos\theta$	$Y = 1/y$	XY	x^2
1	30	.225	$\sqrt{3}/2$	40/9		
2	45	.27	$1/\sqrt{2}$	100/27		
3	60	.32	1/2	100/32		
			2.432	11.273	9.625	2.25

$\therefore Y = a + bX$

$\Sigma Y = na + b\Sigma x \rightarrow 11.273 = 3a + 2.432b$

$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow 9.625 = 2.432a + 2.25b$

بحل المعادلتين $\rightarrow$ $a = 2.342$ $\quad b = 1.746$

③ Fit the curve $y = a + bx$

x	0	0.25	0.5	0.75	1
y	1	1.5	2	2.11	2.718

solution:

i	x	y	xy	x^2
1	0	1		
2	.25	1.5		
3	.5	2		
4	.75	2.11		
5	1	2.718		
Sum				

$a = 1.0564$
$b = 1.6184$

$\therefore Y = a + bx$

" General Form "

$$y = \sum a\,\phi(x)$$

ثوابت — أي دالة

$$\hat{y} = a_0\phi_0(x) + a_1\phi_1(x) + \dots \; a_n\phi_n(x)$$

قام بضرب المعادلة في ϕ_0 $\quad \sum\phi_0 y = a_0\sum\phi_0^2 + a\sum\phi_0\phi_1 + \dots \; a_n\sum\phi_0\phi_n$

~ ~ ~ في ϕ_1 $\quad \sum\phi_1 y = a_0\sum\phi_0\phi_1 + a\sum\phi_1^2 + \dots \; a_n\sum\phi_1\phi_n$

Matrix Form

$$\underbrace{\begin{bmatrix} \; & \; & \; \\ \; & \; & \; \end{bmatrix}_{n\times n}}_{\text{المعاملات}} \underbrace{\begin{bmatrix} \; \\ \; \end{bmatrix}}_{\text{المجاهيل}} = \underbrace{\begin{bmatrix} \; \\ \; \end{bmatrix}}_{\text{الحد المطلق}}$$

- في مصفوفه المعاملات نقوم بوضع كل المعادلات ثم نضرب الصف الاول في ϕ_0 ثم الصف الثاني في ϕ_1 وهكذا بالترتيب
- مصفوفه المجاهيل بنحط المجاهيل
- مصفوفه الحد المطلق نضع الحد المطلق في كل المصفوفات ثم نضرب الصف الاول في ϕ_0 ثم الصف الثاني في ϕ_1 وهكذا بالترتيب

Fit the Curve $\quad y = a_0 + a_1 x + a x^2$

x	0	0.25	0.5	0.75	1
y	1	1.284	1.6487	2.11	2.718

solution: $\quad \phi_0(x) = 1 \quad \phi_1(x) = x \quad \phi_2(x) = x^2$

$\sum\phi_0 = n = 5$

$\sum\phi_0 y = \sum y$

$\sum\phi_0\phi_1 = \sum\phi_1\phi_0$
$= 1$

x	y							ϕ_1	ϕ_2	ϕ_2	ϕ_2^2	$\phi_0 y$	$\phi_1 y$	$\phi_2 y$

Reports :

Fit the curve $y = \frac{x}{ax+b}$

x	0	.25	.5	.75	1
y	1	1.5	2	2.11	2.718

Fit the curve $y = a_0 + a_1 x$

x	1	2	3	4
y	6.5	9.6	13.8	18.3

③ "interpolation with equal spaces"

$\boxed{1}$ shift operator (E)

$$Ef(x) = f(x+h)$$
$$E^{n}f(x) = f(x+nh)$$

$\boxed{2}$ inverse operator (E^{-1})

$$E^{-1}f(x) = f(x-h)$$
$$E^{-n}f(x) = f(x-nh)$$

$\boxed{3}$ forward difference operator (Δ)

$$\Delta f(x) = f(x+h) - f(x)$$

$\boxed{4}$ Backward difference operator (∇)

$$\nabla f(x) = f(x) - f(x-h)$$

$\boxed{5}$ Central difference operator (δ)

$$\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$$
$$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$$

$\boxed{6}$ mean or averaging (μ)

$$\mu f(x) = \frac{1}{2}\, f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)$$
$$\mu = \frac{1}{2}\left[E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right].$$

$\Rightarrow$ prove that

① $\Delta = E\nabla = \nabla E = \delta E^{\frac{1}{2}}$

$$E\nabla = E(1-E^{-1}) = E - \underbrace{\cancel{E}\cancel{E^{-1}}}_{\to 1} = E-1 = \Delta \quad \#$$

$$\nabla E = (1-E^{-1})E = E - E^{-1}E = E-1 = \Delta \quad \#$$

$$\delta E^{\frac{1}{2}} = \left(E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right)E^{\frac{1}{2}} = E - 1 = \Delta \quad \#$$

② $\mu^2 = 1 + \frac{1}{4}\delta^2$

$R.H.S = 1 + \frac{1}{4}\delta^2 \longrightarrow = 1 + \frac{1}{4}\left[E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right]^2$

$= 1 + \frac{1}{4}\left[E - 2 + E^{-1}\right] = 1 + \frac{1}{4}E - \frac{1}{2} + \frac{1}{4}E^{-1} = \frac{1}{4}E + \frac{1}{2} + \frac{1}{4}E^{-1}$

$= \frac{1}{4}\left[E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right]^2 = \left[\frac{1}{2}\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right)\right]^2 = \mu^2$

③ $\Delta = \nabla + \nabla\Delta$

$R.H.S = \nabla(1 + \Delta) \longrightarrow = \nabla(\cancel{1} + E - \cancel{1})$

$= \nabla E \longrightarrow (1 - E^{-1})E = E - 1 = \Delta$ #

④ $\mu\delta = \frac{\Delta + \nabla}{2}$

$\mu\delta = \frac{1}{2}\left[E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right)\left(E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right)$

$= \frac{1}{2}(E - E^{-1}) \longrightarrow \frac{1}{2}(E - E^{-1} + 1 - 1) \longrightarrow \frac{1}{2}\left[(E - 1) + (1 - E^{-1})\right]$

$= \frac{\Delta + \nabla}{2}$

⑤ $\Delta\left(\frac{1}{f_i}\right) = \frac{-\Delta f_i}{f_{i+1} f_i}$

$L.H.S = \frac{1}{f_{i+1}} - \frac{1}{f_i} = \frac{f_i - f_{i+1}}{f_{i+1} f_i} = -\frac{f_{i+1} - f_i}{f_{i+1} f_i}$

$= \frac{-\Delta f_i}{f_{i+1} f_i}$

⑥ $\Delta^n f_i = \nabla^n f_{i+n} = \delta^n f_{i+\frac{n}{2}}$

$\nabla^n f_{i+n} \longrightarrow (1 - E^{-1})^n f_{i+n}$

$(E^{-1})^n (E - 1)^n f_{i+n} \longrightarrow (E-1)^n \cdot E^{-n} f_{i+n}$

$= (E - 1)^n f_i = \Delta^n f_i$ #

$\delta^n f_{i+\frac{n}{2}} = \left(E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right)^n f_{i+\frac{n}{2}} \longrightarrow E^{-\frac{n}{2}}\left[E - 1\right]^n f_{i+\frac{n}{2}}$

$\Delta^n f_i$

⑦ $E = e^{hD}$

$D f(x) = f'(x)$

$D^n f(x) = f^{(n)}(x)$

$Ef(x) = f(x+h)$

Taylor $\longrightarrow f(x+h) = f(x) + h\bar{f}(x) + \frac{h^2}{2!}\bar{\bar{f}}(x) + \frac{h^3}{3!}\bar{\bar{\bar{f}}}(x)$

$f(x+h) = f(x)\left[1 + hD + \frac{h^2D^2}{2!} + \frac{h^3D^3}{3!} + \dots\right)$

$\boxed{\text{Hint}}$ $e^{\square} = 1 + \square + \frac{\square^2}{2!} + \frac{\square^3}{3!} + \dots$

$Ef(x) = e^{hD} f(x) \quad \therefore E = e^{hD}$ #

(8) $D = \frac{1}{h}\left(\Delta - \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} - \dots\right)$

$D = \frac{1}{h}\ln E \quad , E = 1 + \Delta$

$= \frac{1}{h}\ln(1+\Delta)$

$\ln(1+\square) = \square - \frac{\square^2}{2!} + \frac{\square^3}{3!} - \frac{\square^4}{4!} + \dots$

$D = \frac{1}{h}\left(\Delta - \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} - \dots\right)$

(9) $D = \frac{1}{h}\left(\nabla + \frac{\nabla^2}{2!} + \frac{\nabla^3}{3!} + \dots\right)$

$D = \frac{1}{h}\ln E \quad , \nabla = 1 - E^{-1} \; , E^{-1} = 1 - \nabla \longrightarrow E = (1-\nabla)^{-1}$

$D = \frac{1}{h}\ln(1-\nabla)^{-1} \longrightarrow \frac{-1}{h}\ln(1-\nabla) = \frac{-1}{h}\ln(1+(-\nabla))$

$= \frac{-1}{h}\left(-\nabla - \frac{(-\nabla)^2}{2!} + \frac{(-\nabla)^3}{3!} - \dots\right)$

$= \frac{1}{h}\left(\nabla + \frac{\nabla^2}{2!} + \frac{\nabla^3}{3!} - \dots\right)$

(10) $D = \frac{2}{h}\sinh^{-1}\frac{\delta}{2}$

$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}} \longrightarrow E = e^{hD}$

$\delta = e^{\frac{hD}{2}} - e^{-\frac{hD}{2}} \longrightarrow \delta = 2\,\frac{e^{\frac{hD}{2}} - e^{-\frac{hD}{2}}}{2}$

$\delta = 2\sinh\frac{hD}{2} \longrightarrow \frac{\delta}{2} = \sinh\frac{hD}{2}$

$\therefore \frac{2}{h}\sinh^{-1}\frac{\delta}{2} = D$

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// Missing data //

Given $\log 100 = 2$ $\log 101 = 2.0043$ $\log 103 = 2.0128$

$\log 104 = 2.017$ Find $\log 102$ using Δ-operator

sol :

x	100	101	102	103	104
$\log x = y$			α		

نفرض القيمه المفقوده $= \alpha$ ثم نضع $\Delta^n y_0 = 0$

$n+1 =$ عدد خانات الجدول $\longrightarrow \therefore n = 4$

$\Delta^4 y_0 = 0 \longrightarrow (E-1)^4 y_0 = 0$

$(E^4 - 4E^3 + 6E^2 + 4E - 1)\, y_0 = 0$

$y_4 - 4y_3 + 6y_2 + 4y - y_0 = 0$

ويتم التعويض بقيم الجدول في هذه العلاقة

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④ [1] Find $\Delta f_r g_r$ at $x = 4$

$\Delta \frac{f_r}{g_r}$ at $x = 2$ if

x	1	2	3	4	5
f(x)	5	7	10	14	20

X	1	2	3	4	5
g(x)	3	5	8	11	13

sol :

المفروض انتا حافظين :-

- $\Delta f_r g_r = f_r \Delta g_r + g_{r+1} \Delta f_r$

- $\Delta \frac{f_r}{g_r} = \frac{g_r \Delta f_r - f_r \Delta g_r}{g_r \, g_{r+1}}$

$\longrightarrow \Delta (fg)_{x=4} = f_4 \Delta g_r + g_5 \Delta f_4$

$= 14(13-11) + 13(20-14)$ #

$\longrightarrow \Delta \left(\frac{f}{g}\right)_{x=2} = \frac{5(10-7) - 7(8-5)}{8 \times 5}$ #

[2] Determine from the following table polynomial of degree ≤ 5 using newton's method

x	1	2	3	4	5	6
y	14.5	19.5	30.5	53.5	94.5	159.5

← في lagrange لو زودت نقطه مينفعش اكمل حسابات لازم اجيب الحسابات من الاول

← مفيش فروق بين الخطأ في Neton و lagrange كل الموضوع ان Newton شغل الحسابات اسهل وده عن طريق جدول الفروق.

الجدول ده عدد الاعمده فيه بقل (واحد) عن عدد خانات الجدول.

وفيه نكتب على سطر فتسيب سطر

← انا لما بجيب فرق اول عمود بجيب تاني بجيب (ميل)

← باقي الاعمده البسط بيبقى من العمود اللي قبله والمقام. كل عمود بيزيد بيخليني اسيبا $\boxed{X}$

X	y					
1	14.5					
2	19.5					
3	30.5					
4	53.5					
5	94.5					
6	159.5					

$P_5(x) = y_0 + \delta(x - x_0) + \delta^2 (x - x_0)(x - x_1)$

Newton Forward

للحصول على قيمه في اول الجدول

$P = \frac{x - x_0}{h}$

$P(x) = y_0 + P \Delta y_0 + \frac{P(P-1)}{2!} \Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!} \Delta^3 y_0 + \cdots$

← القيم الموجوده في اول كل عمود

Newton Backward

للحصول على قيمه في اخر الجدول

$P = \frac{x - x_n}{h}$

$P(x) = y_n + P \nabla y_n + \frac{P(P+1)}{2!} \nabla^2 y_n + \frac{P(P+1)(P+2)}{3!} \nabla^3 y_n + \cdots$

← القيم الموجوده في اخر كل عمود

Gaussian Forward

للحصول على قيمة في نصف الجدول

$$P = \frac{X - x_0}{h}$$ ← اقرب x في الجدول للـ x المطلوبة

$$P(X) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_{-1} + \frac{(P+1)P(P-1)}{3!}\Delta^3 y_{-1} + \frac{(P+1)P(P-1)(P-2)}{4!}\Delta^4 y_{-2} + \frac{(P+2)(P+1)P(P-1)(P-2)}{5!}\Delta^5 y_{-2}$$

$$x_0 \to y_0 \searrow \Delta y_0 \nearrow \Delta^2 y_{-1} \searrow \Delta^3 y_{-1} \nearrow \Delta^4 y_{-2} \searrow \Delta^5 y_{-2}$$

Gaussian Backward

للحصول على قيمة في نصف الجدول

$$P(X) = y_0 + P\Delta y_{-1} + \frac{(P+1)P}{2!}\Delta^2 y_{-1} + \frac{(P+1)P(P-1)}{3!}\Delta^3 y_{-2} + \frac{(P+2)(P+1)P(P-1)}{4!}\Delta^4 y_{-2} + \frac{(P+2)(P+1)P(P-1)(P-2)}{5!}\Delta^5 y_{-3} + \cdots$$

$$x_0 \to y_0 \nearrow \Delta y_{-1} \searrow \Delta^2 y_{-1} \nearrow \Delta^3 y_{\cancel{-2}} \searrow \Delta^4 y_{-2}$$

Strling

للحصول على قيمة في نصف الجدول

$$S = \frac{1}{2}\left[GF + GB\right]$$

From the following table Find the number of students who obtained Less than 45 and 75 mark

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
no-of student	31	42	51	35	31

Sol : انا مش بتعامل مع فترات بتعامل مع Points يبقى هحول الجدول

لتقدم. مفتاح التحويل هو كلمة (Less than) في رأس السؤال

Marks	40	50	60	70	80
no.of st.	31	73	124	159	190

x	y				
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		41			
80	190				

$$P(X) = 31 + 0.5(42) + \frac{(.5)(.5-1)}{2!}(9) + \frac{(.5)(.5-1)(.5-2)}{3!}(-25) + \frac{(.5)*(.5-1)(.5-2)(.5-3)}{4!}(\quad)$$

$\therefore$ no. of st =

$$P(x) = 190 + (.5)(31) + \frac{(.5)(-.5+1)}{2!}(-4) + \frac{(-.5)(-.5+1)(-.5+3)}{3!}(12)$$

$$+ \frac{(.5)(-.5+1)(-5+2)(\quad)}{4!}(\quad)$$

no. of st =

From the following table find $f(2.3)$

x	0	1	2	3	4	5
y	-3	3	11	27	57	107

Sol :

G.F	G.B	S
$P(x) = 11 + (0.3)(16) + \frac{(.3)(.3-1)}{2!}(8) + \frac{(0.3+1)(.3)(.3-1)}{3!}(6) = 14.86$	$P(x) = 11 + 0.3(8) + \frac{(.3+1)(.3)}{2!}(8) + \frac{(0.3+1)(0.3)(0.3-1)}{3!}(6) = 14.86$	

x	y					
0	−3					
		6				
1	3		2			
		8		6		
x_0 2	y_0 11		8		6	
		16		6		0
3	27		14		0	
		30		6		
4	57		20			
		50				
5	107					

⑥ Numerical differentiation

$$F(x+h) = f(x) + h\bar{f}(x) + \frac{h^2}{2!}\bar{\bar{f}}(x) + \cdots$$

$$f(x-h) = f(x) - h\bar{f}(x) + \frac{h^2}{2!}\bar{\bar{f}}(x) - \cdots$$

مفكوك تايلور ؟ مهمين حتى باقي الـ corse

1st derivative $\bar{f}(x)$

** Forward

$$\bar{f}(x) = \frac{f(x+h) - f(x)}{h}$$

$$|T.E| \leq \frac{h}{2}|\bar{\bar{f}}(\xi)| \rightarrow [x, x+h]$$ اكبر قيمه للمشتقه الثانيه في الفتره

** Backward

$$\bar{f}(x) = \frac{f(x) - f(x-h)}{h}$$

$$|T.E| \leq \frac{h}{2}|\bar{\bar{f}}(\xi)| \rightarrow [x-h, x]$$ اكبر قيمه للمشتقه الثانيه في الفتره

** Central

$$\bar{f}(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$|T.E| \leq \frac{h^2}{2!}|f^{(3)}(\xi)| \rightarrow [x-h, x+h]$$ اكبر قيمه للمشتقه الثالثه في الفتره

** Richardson

$$\bar{f}(x) = \frac{4}{3}\phi\left(\frac{h}{2}\right) - \frac{1}{3}\phi(h)$$

$$\phi(h) = \frac{f(x+h) - f(x-h)}{2h}$$

$$\phi\left(\frac{h}{2}\right) = \frac{f(x+\frac{h}{2}) - f(x-\frac{h}{2})}{h}$$

Error $\rightarrow O(h^4)$

أخد بالك الـ error بيتناسب مع (h) أس كام لانه في الامتحان ممكن ميقوليش اسم الطريقه ويجيب الـ error بيتناسب مع h اس كام

** Repeated Richardson

$$D_{n,1} = \phi\left(\frac{h}{2^{n-1}}\right) \quad n = 1, 2, \ldots$$

$$D_{n,m+1} = \frac{4^m}{4^m - 1} D_{n,m} - \frac{1}{4^m - 1} D_{n-1,m}$$

Error $\rightarrow O(h^{2n+2})$ عدد مرات التكرار

2^{nd} derivative

$\times\times$ Central

$$\bar{\bar{f}}(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

$$|T.E| \leq \frac{h^2}{12} \left| f^{(4)}(s) \right| \longrightarrow [x-h, x+h]$$ اكبر قيمه للمشتقه الرابعه ف الفتره

→ if $f(x) = e^x$ approximate $\bar{f}(1)$ using $h = 0.1$ by:

① F.W ② B.W

③ central , and find T.E in each cass

⇝ F.W

$$\bar{f}(x) = \frac{f(x+h) - f(x)}{h} , \quad \bar{f}(1) = \frac{f(1.1) - f(1)}{0.1}$$

$$= 2.8588$$

$$T.E \leq \frac{h}{2} \left| \bar{\bar{f}}(s) \right| \longrightarrow [1, 1.1]$$

اخد بالك ان عندي (1.1) القيمه المطلقه لها ياخد الاكبر مشتقه

يعني لو عند مثلا 2 & 3 - يبقى هاخد (3)

$$\bar{\bar{f}}(x) = e^x \longrightarrow \bar{\bar{f}}(s) = e^{1.1}$$

⇝ B.W

$$\bar{f}(1) = \frac{f(1) - f(0.9)}{0.1} = 2.58$$

$$\text{Error} \leq \frac{h}{2} \left| \bar{\bar{f}}(s) \right|^{0.1} \longrightarrow [x-h, x] = [0.9, 1]$$

$$\bar{\bar{f}}(s) = e , \quad |T.E| \leq .0.135$$

⇝ Central

$$\bar{f}(1) = \frac{f(1.1) - f(.9)}{.2} = 2.77$$

$$T.E \leq .0005$$

→ Using the following data to find $\bar{f}(6)$ with error $= O(h)$

x	6	6.1	6.2	6.3	6.4
y	-.175	-.1998	-.223	-.2422	-.2596

→ B.W does n't work لانه هو عايز $\bar{f}(6)$ فلازم ارجع ورا خطوه وكده مش هينفع

$$\bar{f}(6) = \frac{f(6.1) - f(6)}{0.1}$$

$$= -.248$$

$\rightarrow$ Find $\bar{F}(1.3)$, $h = 0.5$ for $f(x) = \sqrt{x^2+1}$ by Richardson with $n = 3$

ـــه اول ما يذكر $n=\square$ يبقى بشتغل بـ Reapeted Richardson

$$D_{n,1} = \phi\left(\frac{h}{2^{n-1}}\right) \quad n = 1, 2, 3$$

$$D_{11} = \phi h = \frac{f(x+h) - f(x-h)}{2h}$$

$$D_{21} = \phi\left(\frac{h}{2}\right) = \frac{f(x+\frac{h}{2}) - f(x-\frac{h}{2})}{h}$$

$$D_{31} = \phi\left(\frac{h}{4}\right) = \frac{f(x+\frac{h}{4}) - f(x-\frac{h}{4})}{h/2}$$

$\frac{4}{3}$, $\frac{-1}{3}$	$\frac{16}{15}$, $\frac{-1}{15}$	
$D_{11} = 0.778$		
	$D_{22} = 0.798$	
$D_{21} = 0.789$		$D_{33} = 0.793$
	$D_{32} = 0.793$	
$D_{31} = 0.792$		

$$D_{22} = \frac{-1}{3} D_{11} + \frac{4}{3} D_{21}$$

$$D_{32} = \frac{-1}{3} D_{21} + \frac{4}{3} D_{31}$$

$$D_{33} = \frac{-1}{15} D_{22} + \frac{16}{15} D_{32}$$

$$T.E = O(h^8)$$

7

Numerical integration

$$I = \int_a^b f(x)\,dx$$

① Trapezoidal Valid for any (n)

$$I = \frac{h}{2}\left(y_0 + y_n + 2(y_1 + y_2 + y_3 + \dots)\right)$$

$$I = \frac{h}{2}\left[\text{الاول} + \text{لاخير} + 2(\text{الباقى})\right]$$

$$|T.E| \leq \frac{(b-a)^3}{12n^2} M_2$$

$[a,b]$ اكبر قيمه للمشتقه الثانيه فى الفترة

$\rightarrow \boxed{\frac{b-a}{n} = h}$

② Simpson Valid for even (n)

$$I = \frac{h}{3}\left(y_0 + y_n + 2(y_2 + y_4 + \dots) + 4(y_1 + y_3 + \dots)\right)$$

$$I = \frac{h}{3}\left[\text{الاول} + \text{الاخير} + 2(\text{الزوجى}) + 4(\text{الفردى})\right]$$

$y_0 \; y_1 \; y_2 \; y_3$

$$|T.E| \leq \frac{(b-a)^5}{180n^4}$$

$[a, b]$ اكبر قيمه للمشتقه الرابعه غ الفترة

③ Weddle

1 5 1 6 1 5 1

$n = 6$
$$I = \frac{3h}{10}\left(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right)$$

$n = 12$

1 5 1 6 1 5 1
1 5 1 6 1 5 1

$$I = \frac{3h}{10}\left(y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + 2y_6 + 5y_7 + y_8 + 6y_9 + y_{10} + 5y_{11} + y_{12}\right)$$

③ Gaussian

$$\int^{1} f(x)\,dx = \sum w_i f(x_i)$$

** Two points

$$I = w_1 f(x_1) + w_2 f(x_2)$$

$$w_1 = w_2 = 1$$

$x_1 = 1/\sqrt{3}$ $x_2 = -1/\sqrt{3}$

$$I = f\left(\frac{1}{\sqrt{3}}\right) + f\left(\frac{-1}{\sqrt{3}}\right)$$

الجزء ده اذا كانت حدود التكامل ليست
من -1 ,1 واجبرنا نشتغل بـ Gauss

** Three points

$$I = W_1 f(x_1) + W_2 f(x_2) + W_3 f(x_3)$$

$$W_1 = W_3 = \frac{5}{9} \qquad W_2 = \frac{8}{9}$$

$$x_1 = \sqrt{\frac{3}{5}} \qquad x_2 = 0 \qquad x_3 = -\sqrt{\frac{3}{5}}$$

$$I = \frac{5}{9}\left(f\left(\sqrt{\tfrac{3}{5}}\right) + f\left(-\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9} f(0)$$

$$\int_a^b f(x)\,dx \rightarrow \int_{-1}^{1} f(z)\,dZ$$

$$\frac{x-a}{b-a} = \frac{Z+1}{2} \rightarrow$$

$$dx = \frac{(b-a)}{2}\,dZ$$

⑤ Romberg

$$R_{11} = \frac{b-a}{2}\left[F(a) + F(b)\right]$$

$$R_{21} = \frac{R_{11}}{2} + \frac{b-a}{2}\left[f\left(a + \frac{b-a}{2}\right)\right]$$

$$R_{31} = \frac{R_{21}}{2} + \frac{b-a}{4}\left[f\left(a + \frac{b-a}{4}\right) + f\left(a + \frac{3(b-a)}{4}\right)\right]$$

تم باقي الحل مثل Richardson

→ Find approximate value to $\int_0^6 2x\,dx$ using:

① Trapezoidal
② simpson
③ weddle

SoL: Let $n = 6$, $h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6

→ Find the no. of $\boxed{\text{sub intervals}}$ n to approximate $\int_0^1 \frac{1}{x^2+6}\,dx$ with error Less than 10^{-4} using Trapezoidal

sol : $\frac{(b-a)^3}{12n^2} M_2 \leq 10^{-4}$

$F(x) = \frac{1}{x^2+6} \longrightarrow \overset{=}{F}(x) = \frac{6x-12}{(x^2+6)^3}$

$\overset{=}{F}(0) = -12/6^3$, $\overset{=}{F}(1) = -6/7^3$ $\therefore M = 12/6^3$

$\rightarrow \frac{1}{12n^2}(0.056) \leq 10^{-4} \longrightarrow \frac{12n^2}{0.056} \geq 10^4 \longrightarrow n \geq 6.8 \therefore n \geq 7$

→ Find approximate Value to $\int_1^3 \frac{1}{x}\,dx$ using Romberge with $k=3$ compare with exact sol.

sol: $R_{11} = \frac{b-a}{2}\left[f(a)+f(b)\right] = \frac{3-1}{2}\left[F(1)+F(3)\right] = 1.333$

$R_{21} = \frac{R_{11}}{2} + \frac{b-a}{2}\left[f\left(a+\frac{b-a}{2}\right)\right] = 1.168$

$R_{31} = \frac{R_{21}}{2} + \frac{b-a}{4}\left[F\left(a+\frac{b-a}{4}\right) + F\left(a+\frac{3(b-a)}{4}\right)\right] = 1.117$

exact sol ⇒			
$I = \ln x \Big\vert_1^3 = \ln 3$	R_{11}		
$= 1.8993$	R_{21}	$R_{22} = 1.112$	
$\therefore$ Error $= \lvert 1.8993 - 1.0993 \rvert$	R_{31}	$R_{32} = 1.1001$	$R_{33} = 1.0993$

→ Use 3- Points Gaussian to evaluate ,

① $\int_{-1}^{1} 2/\sqrt{x^2+4}\;dx$ ② $\int_0^1 \ln(3+\sin x)\,dx$

① $I = \frac{5}{9}\left[f\left(\sqrt{\tfrac{3}{5}}\right) + F\left(-\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9} f(0)\Big] = 2.016$

$f\left(\sqrt{\tfrac{3}{5}}\right) = 2/\sqrt{\tfrac{3}{5}+4}$

$f\left(-\sqrt{\tfrac{3}{5}}\right) = 2/\sqrt{\tfrac{3}{5}+4}$

$f(0) = \frac{2}{\sqrt{4}}$

Scanned by CamScanner

② $\frac{x-a}{b-a} = \frac{Z+1}{2}$

$x = \frac{Z+1}{2}$, $dx = \frac{dZ}{2}$

$$\int_{-1}^{1} \ln\left(3 + \sin\left(\frac{Z+1}{2}\right)\right) \frac{dZ}{2}$$

$$\frac{5}{9}\left[F\left(\sqrt{\tfrac{3}{5}}\right) + F\left(-\sqrt{\tfrac{3}{5}}\right)\right] + \frac{8}{9} F(0)$$

$$F\left(\sqrt{\tfrac{3}{5}}\right) = \ln\left(3 + \sin\left(\frac{\sqrt{\frac{3}{5}}+1}{2}\right)\right)$$

$I = 1.2385$

Report:

① How small must be "h" in order to compute the integral $I = \int_{2}^{11} e^{2x}\, dx$ by simpson's ruLe with an accuracy of 10^{-4}

ng 2- subintervals of 19

he estimate for the same

Vals is?

$f(11) + 2f(5)]$

$) - f(11) + 2f(5)]$

② the value $\int f(x)\, dx$ by usin

simpson $\frac{}{3}$ is 702.039 . T

integration using 4- subinter

a) $702.039 + \frac{8}{3}[2f(7) -$

b) $\frac{702.039}{2} + \frac{8}{3}[2f(7$

8

Numerical sol. of ODE with IV

Given: $\bar{y} = f(x,y)$, $y(x_0) = y_0$, $h = \frac{x_n - x_0}{n}$

$$\begin{array}{ccccc} x_0 & x_1 & x_2 & \cdots & x_n \\ y_0 = \checkmark & y_1 = ? & y_2 = ? & \cdots & y_n = ? \end{array}$$

(one step)

اسمها one step لانه كل نقطه بتعتمد على اللى قبلها

1 Euler's method

$$y_{i+1} = y_i + h\underbrace{f(x_i, y_i)}_{\bar{y}}$$

ده الحدود الاولى من مفكوك تايلور

• Global error الخطأ التراكمى

$$|e_n| \leq \frac{hK}{2M}\left[(1+hM)^n - 1\right] \quad , n = 1, 2 \cdots \Rightarrow e_1, e_2, \cdots$$

local error الخطأ المحلى

ـ الخطأ عند نقطه معينه حيث يتم التعويض بـ n رتبة النقطه المراد حسابا الخطأ عندها

2 Taylor method

** 2^{nd} order

$$y_{i+1} = y_i + hf_i + \frac{h^2}{2!} f_i$$

** 3^{rd} order

$$y_{i+1} = y_i + hf_i + \frac{h^2}{2!} \bar{f_i} + \frac{h^3}{3!} f_i''$$

" مفكوك تايلور " $y_{i+1} = y_i + h\bar{y}_i + \frac{h^2}{2!} + \frac{h^3}{3!} y_i''' + \frac{h^4}{4!} y_i'''' + \cdots$

3 4^{th} order RK (Runge-Kutta)

$$y_{i+1} = y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$K_1 = f(x_i, y_i)$$
$$K_2 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}K_1)$$
$$K_3 = f(x_i + \frac{h}{2}, y_i + \frac{h}{2}K_2)$$
$$K_4 = f(x_i + h, y_i + hK_3)$$

4 Modified Euler (corrector-predictor)

ده نفس Euler الاولى ولكن بحسن فى y_1 علشان ناقل الخطأ بحسن لحد ما القيمه تثبت معايا تكرر فى الامتحان هحسب 4 مرات بس

Predictor step

$$y^p_{i+1} = y_i + hf(x_i, y_i) = y^{c(0)}_{i+1}$$

Corrector step $\rightarrow$ (4) $n = 0, 1, 2, 3$

$$y^{c(r+1)}_{i+1} = y_i + \frac{h}{2}\left[f(x_i, y_i) + f(x_{i+1}, y^{c(r)}_{i+1})\right]$$

⊛ system of 1^{st} order ODE

2^{nd} order ODE $\longrightarrow$ system of 1^{st} order $\Rightarrow$ solve

① Approximate sol. of ODE. $\bar{y} - y + t^2 - 1 = 0$, $0 \leq t \leq 0.6$

$y(0) = 0.5$. using

a) Euler

b) 2nd & 3rd order Taylor

c) 4th order RK (taking one step) $y|_{t=0.3}$

d) obtain global error of method (a) with $h = 0.2$ in interval $t \in [0, 2]$, knowing that exat sol. is $y(t) = (t+1)^2 - 0.5e^{t}$.

① $\bar{y} = f(t, y) \longrightarrow \bar{y} = y - t^2 + 1 \rightsquigarrow f(t,y) = y - t^2 + 1$

② $h = 0.3$ علشان ع المطلوب التالت هو طالب step واحده وعايز اقف عند (.3)

$t_0 = 0$, $y_0 = .5$; $t_1 = 0.3$, $y_1 = ?$; $t_2 = 0.6$, $y_2 = ?$

③ a) Euler $\longrightarrow y_{i+1} = y_i + h\,f(t_i, y_i) \longrightarrow y_{i+1} = y_i + 0.3(y_i - t_i^2 + 1)$

$i = 0 \qquad y_1 = y_0 + 0.3(y_0 - t_0^2 + 1) = 0.95$

$i = 1 \qquad y_2 = y_1 + 0.3(y_1 - t_1^2 + 1) = 1.508$

b) 2nd order Taylor $\longrightarrow y_{i+1} = y_i + h f_i + \frac{h^2}{2!}\bar{f}_i$

$\bar{f}_i = \bar{y} - 2t = y - 2t - t^2 + 1$

$i = 0 \qquad y_1 = y_0 + 0.3 f_0 + \frac{0.09}{2}\bar{f}_0 = 1.0175$

$i = 1 \qquad y_2 = y_1 + 0.3 f_1 + \frac{0.09}{2}\bar{f}_1 = 1.6558$

** 3rd order $\longrightarrow y_{i+1} = y_i + h f_i + \frac{h^2}{2!}\bar{f}_i + \frac{h^3}{3!}\bar{\bar{f}}_i$

$\bar{\bar{f}}_i = \bar{y} - 2 - 2t = y - t^2 - 2t - 1$

$i = 0 \qquad y_1 = y_0 + h f_0 + \frac{h^2}{2!}\bar{f}_0 + \frac{h^3}{3!}f_0''$

دى الـ initial لكل المسأله واحده يعني y_0 دي لكل $= .5$ لكن مينفعش أخد y_1 من Euler لـ y_1 لـ 2nd

$= 1.0175 + \frac{(0.3)^3}{6}(y_0 - t_0^2 - 2t_0 - 1) = 1.01525$

$i = 1 \qquad y_2 = y_1 + h f_1 + \frac{h^2}{2!}\bar{f}_1 + \frac{h^3}{3!}\bar{\bar{f}}_1$

c) RK $\longrightarrow$ $y_{i+1} = y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

one step أخد بالك حبيبي عند $i=0$ بس لأنه قال

$i = 0$ $\qquad y_1 = y_0 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = f(t_0, y_0) = y_0 - t_0^3 + 1 = 1.5$

$k_2 = f(t_0 + \frac{h}{2}, y_0 + \frac{h}{2}k_1) = f(0.15, 0.725)$

0 .3 .5 .15 1.5

$k_2 = 0.725 - (0.15)^2 + 1 = 1.7025$

$k_3 = f(t_0 + \frac{h}{2}, y_0 + \frac{h}{2}k_2) = f(0.15, 0.7553)$

$= 1.7328$

$k_4 = f(t_0 + h, y_0 + hk_3) = f(0.3, 0.156)$

$= 1.929$

$\therefore y_1 = 1.01503$

d) $|e_n| \leqslant \frac{hK}{2M}[(1 + hM)^n - 1]$

$K \longrightarrow Max|\bar{\bar{y}}|$

$M \longrightarrow Max|f_y|$ $\longrightarrow$ تفاضل الدالة تفاضل جزئي بالنسبة لـ y

$\bar{\bar{y}}$ ← اما اجيبها in exact اللي محصول وأفاضلها مرتين ودة الأفضل لأنه لا يعتمد على قيمة y. أما اني اخد معادلة $\bar{y}$ السابقة وأفاضلها مرة كمان واشتغل بيها لكن ملاقيها بتعتمد على قيمة y يعني مضطر احسب y عند 2

From exact $\longrightarrow \bar{y} = 2(t+1) - 0.5e^t$

$\bar{\bar{y}} = 2 - 0.5e^t$

$\bar{\bar{y}} = 1.5$

$\hookrightarrow = -1.67$

$K = |2 - 0.e^t| = 1.67$

$M = 1$

$|e_n| \leqslant \frac{0.2(1.67)}{2}(1 + 0.2)^n - 1$

② use modified Euler to approximate $y(0.3)$ if

$\bar{y} = x + y + 1 \qquad y(0) = 0$

① $y' = f(x, y) = x + y + 1$

② $x_0 = 0$, $y_0 = 0$ ⟶ $x_1 = 0.3$, $y_1 = ?$

→ Predict step

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = 0.3$$

دي متنقلش step ايد i يعني اللي تتغير هنا هو r بس لكن i ثابتة

→ Correct

$$y_{i+1}^{c(r+1)} = y_i + \frac{h}{2}\left[f(x_i, y_i) + f(x_{i+1}, y_{i+1}^{c(r)})\right]$$

** r_0

$$y_1^{c(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{c(0)})\right]$$

دة يعني y_1 اللي لسه معملتلهاش تصحيح

$$y_1^{c(1)} = 0.39$$

** r_1

$$y_1^{c(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1 + y_1^{c(1)})\right]$$

$$y_1^{c(2)} = 0.4035$$

$$y_1^{c(3)} = 0.4059$$

$$y_1^{c(4)} = 0.4058$$